Noise Analysis in functional MRI using Time Varying Autoregressive Models

Chris J. Long[1], Emery N. Brown[2,4], Christina Triantafyllou[1], Larry L. Wald[1], Victor Solo[1,3]
[1]MGH/MIT/HMS Martinos Center for Biomedical Imaging, Charlestown, MA.
[2]Dept. of Anesthesia & Critical Care, Massachusetts General Hospital, Boston, MA.
[3]Dept. of Statistics & Dept of Computer & Electrical Engineering, University of Michigan
[4]Harvard Medical School/MIT Division of Health Sciences & Technology, Cambridge, MA.
Email : cjl@nmr.mgh.harvard.edu

INTRODUCTION

An important issue in functional MRI (fMRI) analysis is an accurate characterization of the noise processes present in the data. Conventional fMRI noise representations often assume stationarity (or time-invariance) in the noise generating sources, but the fixed nature of these assumptions may suppress important dynamic information about brain function. We present a new time-varying procedure for examining noise structure in fMRI data. Specifically, we derive a local parametric model (AR+drift) which tracks the temporal covariance by allowing dynamic evolution of the model parameters. The main idea is to approximate complex nonstationary behaviour by means of a collection of simple but numerous parametric models. Prior to exploring time variation in these parameters, window-widths that are well-suited to the latent time-varying noise structure need to be identified. Since these underlying time-varying characteristics are not known in practice, it is necessary to determine the window-widths from the measured timeseries. This is accomplished through the introduction of a new window-width selection mechanism based on Stein's Unbiased Risk Estimator (SURE). We examine the behaviour of our time-varying method on simulated data and measure its whitening performance by analysing resting data acquired from 1.5, 3 and 7T magnetic field strengths. Incorporation of time variation in the AR parameters seems to lead to an overall decrease in the level of residual structure in the data.

BACKGROUND & METHODS

- ***What is meant by nonstationarity?*** Statistically speaking, a nonstationary process is taken to mean a process whose covariance *K(t,s)* is a function of *t* and *s*. This contrasts to the stationary situation where *K(t,s)* depends on *t-s*.
- **Stationary & Nonstationary Aspects of fMRI Noise**
  - ***Machine related and nuisance physiological components.*** E.g. stochastic (white) background noise deriving from nonlinearities and thermal fluctuations in the system electronics and slow drifts caused by small deviations in the scanner magnetic field. Components related to periodic small-scale pulsations in the brain deriving e.g. from heart rate, subject respiration and fluctuations caused by unresolved subject motion. Aliasing. (*Stationary (with the exception of trend but this is simple to model out separately) & uninteresting*).
  - ***TE-dependent physiological noise.*** In the absence of constraining experimental stimuli, the net measured blood-deoxygenation contrast may contain variance originating from nonstationary background (uncontrolled) neuronal activity. This time-varying behaviour is induced by neurophysiological factors including variation in the numbers of contributing neurons at a given time instant, the accumulative effects of differences between respective neuronal firing rates and other neural variability associated with extraneous auditory and visual stimuli or background memory processes. (*Nonstationary & of greater interest*).
- **A time-varying autoregressive (TVARM) model for functional MRI noise.**
  We build on Local Kernel methods to model these effects. See e.g. [1]
  We combine a local mean estimation with a time-varying q^{th} order autoregressive process

parameterised by the weightings, $\{a_r\}_{r=1}^{q}$. The local model for each time series y in the neighborhood h of t_0 may be written

$$y_{t,p} = m_{t,p} + w_{t,p} \qquad t \in t_0 \pm u/2$$ # (Equation(1))

where $w_{t,p}$ is a Gaussian autoregressive noise at pixel p with q^{th} –order serial autocorrelation $w_{t,p} = \sum_{r=1}^{q} a_r w_{t-r,p} + \epsilon_{t,p}$.

u is the effective kernel width, $u = 2Th - 1$, T is the signal length and m_p is the local mean. Subsequently,

$$y_{t,p} = m_p\Big(1 - \sum_{r=1}^{q} a_r\Big) + \sum_{r=1}^{q} a_r y_{t-r,p} + \epsilon_{t,p} \qquad t \in t_0 \pm u/2$$ # (Equation(2))

$$= c_p + \sum_{r=1}^{q} a_r y_{t-r,p} + \epsilon_{t,p}$$

where

$$c_p = m_p\left(1 - \sum_{r=1}^{q} a_r\right)$$ # (Equation(3))

Eq. (2) may be fit over the locally windowed neighborhood by minimising the following local likelihood in the locale of t_0

$$\min_{\theta_{t_0,p}} \sum_{t=q}^{N-1}\left(y_{t,p} - c_p - \sum_{r=1}^{q} a_r y_{t-r,p}\right)^2 W\left(\frac{t-t_0}{h}\right)$$ # (Equation(4))

where $\theta_{t,p} = (c_p, a_1, \ldots, a_r)$, is the vector of unknown parameters, and the windowing function $W_{\frac{t-t_0}{h}}$ is constructed so that all the data points contributing to the estimation are smoothly weighted in proportion to their distance from t_0. In this work we have chosen W as the Epanechnikov kernel.

The SURE criterion approximates the risk for a given window-width by combining the goodness of model fit with a model complexity term. That is

$$RISK = (data - model)\ discrepancy + model\ complexity$$

$$SURE(h_p) = \log_e\left(\frac{\|\hat{e}_p\|_2^2}{T}\right) + \frac{2}{h_p T}$$

RESULTS

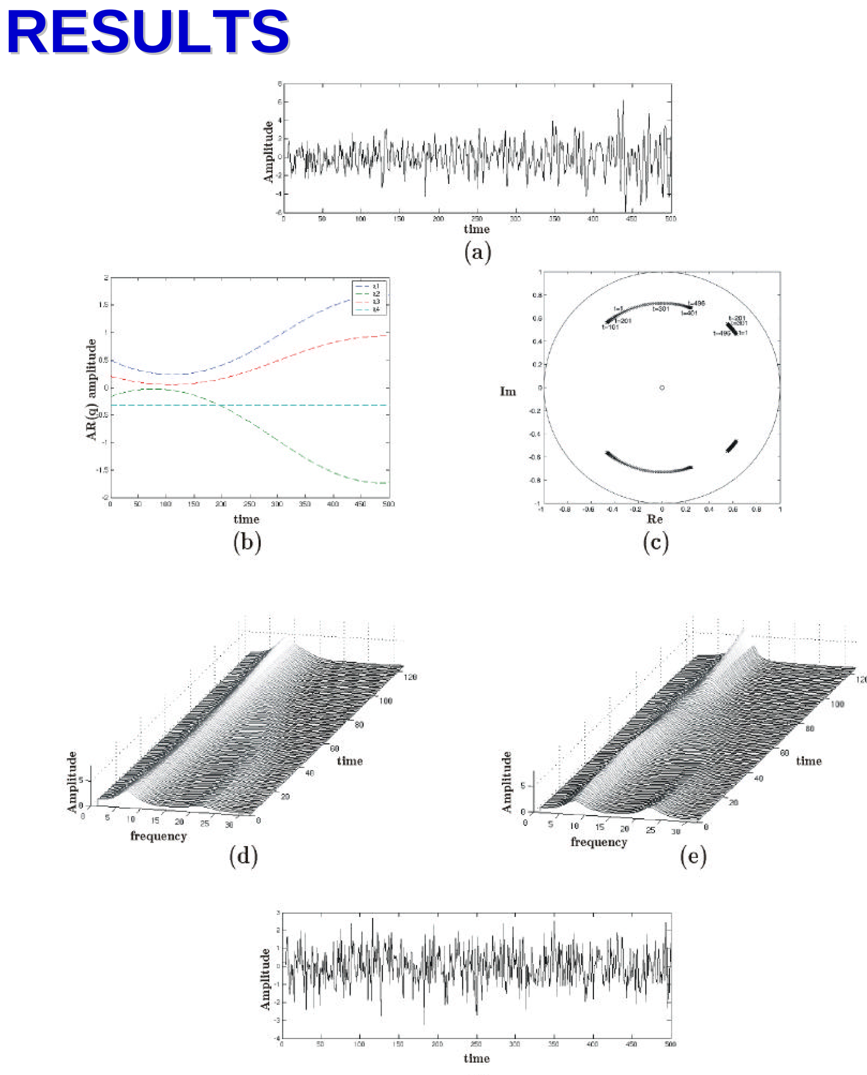

Figure 1

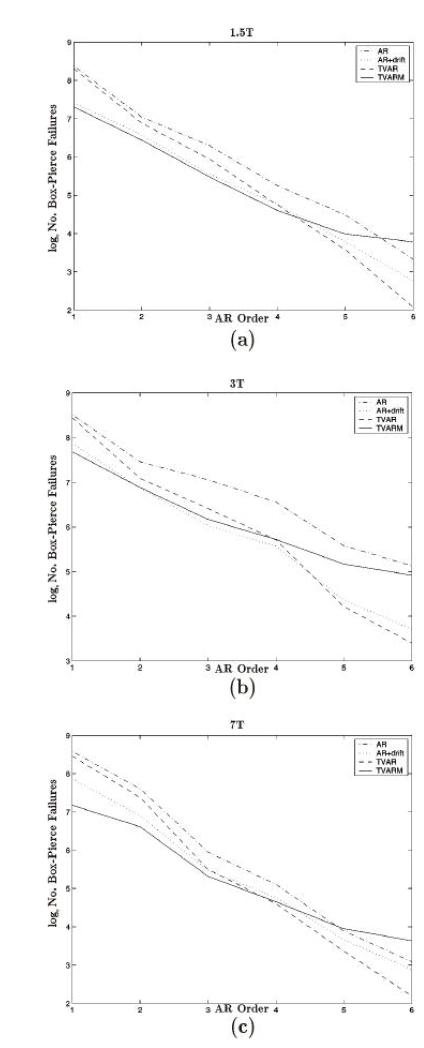

Figure 2

Figure 1: Time-varying AR(4) simulated example. (a) Shows a timeseries realisation from a simulated time-varying system (b) plots of the four AR coefficients against time (c) complex-conjugate pole-pairs from the AR(4) polynomial, showing their evolution with time and the region of stability (unit circle) (d) & (e) instantaneous power spectral density plots respectively of theoretical and recovered AR(4) parameters using TVAR. (f) whitened residuals from the TVAR(4) fit.

Figure 2: Illustrating the respective whitening of four competing noise modelling methods on human resting data acquired on the 1.5, 3 and 7T Siemens systems at MGH. The four methods correspond to stationary (AR(q) & AR(q) + drift) and two nonstationary (TVAR(q) & TVARM(q)).

CONCLUSION

TVARM procedures naturally extend parametric stationary AR estimations and provide related but perhaps richer information about fMRI noise structure. Building temporal flexibility into the autoregressive noise model yields improvement over its stationary counterpart, and this was most pronounced at low model orders.

REFERENCES:

1. Fan & Gijbels, Chapman&Hall,1996.

ACKNOWLEDGEMENTS:
This work was supported by NIH grants NCRR P41 RR14705 to the Center for Functional Imaging Technologies at Massachusetts General Hospital, NIMH K02 MH61637 and NIBIB R01 EB0522

THE MIND INSTITUTE