Time dependent regularization for functional magnetic resonance inverse imaging

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Outline of the E-Poster

- Introduction
  - Inverse Imaging (InI): ultrafast functional MRI.
  - Ill-posed image reconstruction -> regularization.
- Methods
  - Statistical approach to InI inverse problem.
  - Type II maximum likelihood regularization.
- Results
  - Estimates for static and dynamic regularization.
- Summary

Introduction

Why Inverse Imaging (InI)?

- Dynamic imaging with fast temporal sampling
  - Whole head reconstruction with 100 ms sampling.
  - Enhanced temporal mapping of hemodynamics.

- Potential applications of InI
  - Studying hemodynamic response properties across brain areas.
  - Neuronal timing information in fMRI data.

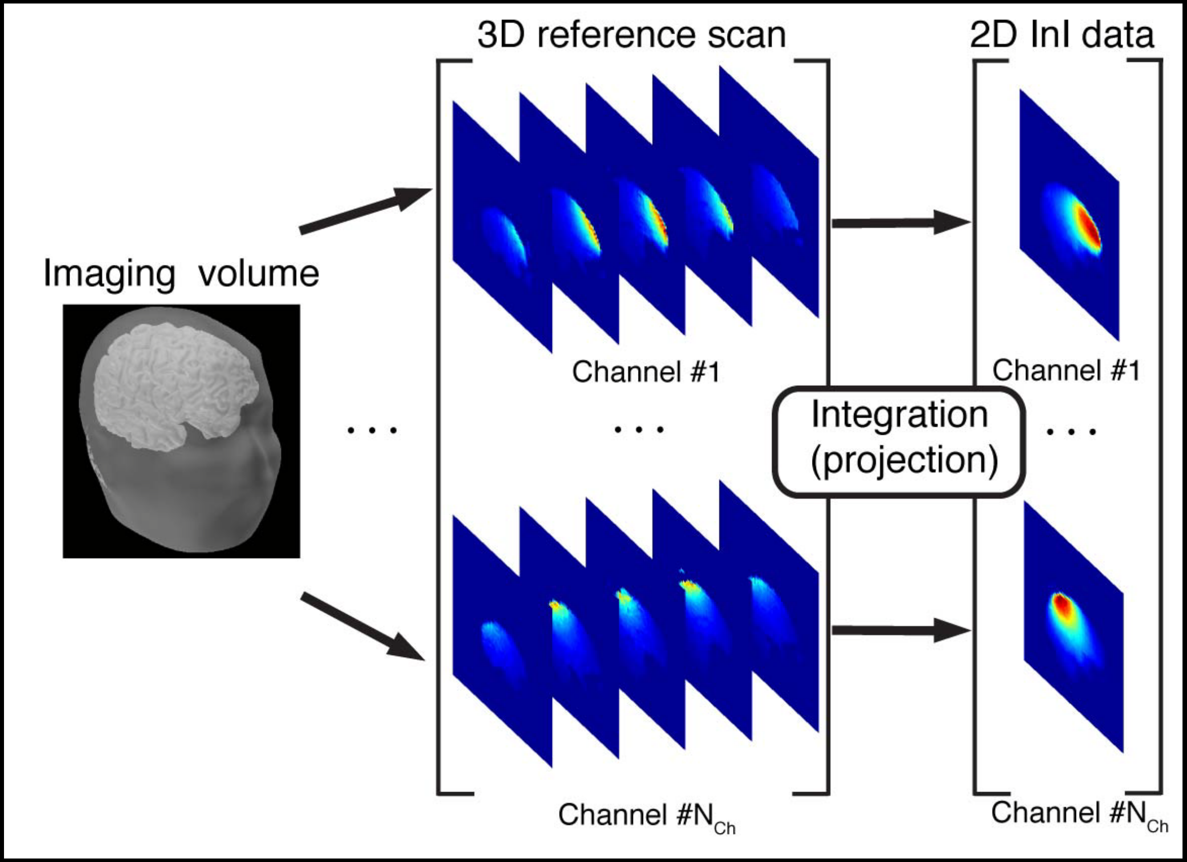
How InI is done

- Phase encoding omitted along some axis:
  - Parallel imaging recon with coil sensitivity only.
  - Non-uniform spatial resolution.

- Image reconstruction:
  - Requires solving an ill-posed inverse problem.
  - Necessitates regularization to obtain stability.
  - Inverse problem introduces spatial ambiguity.

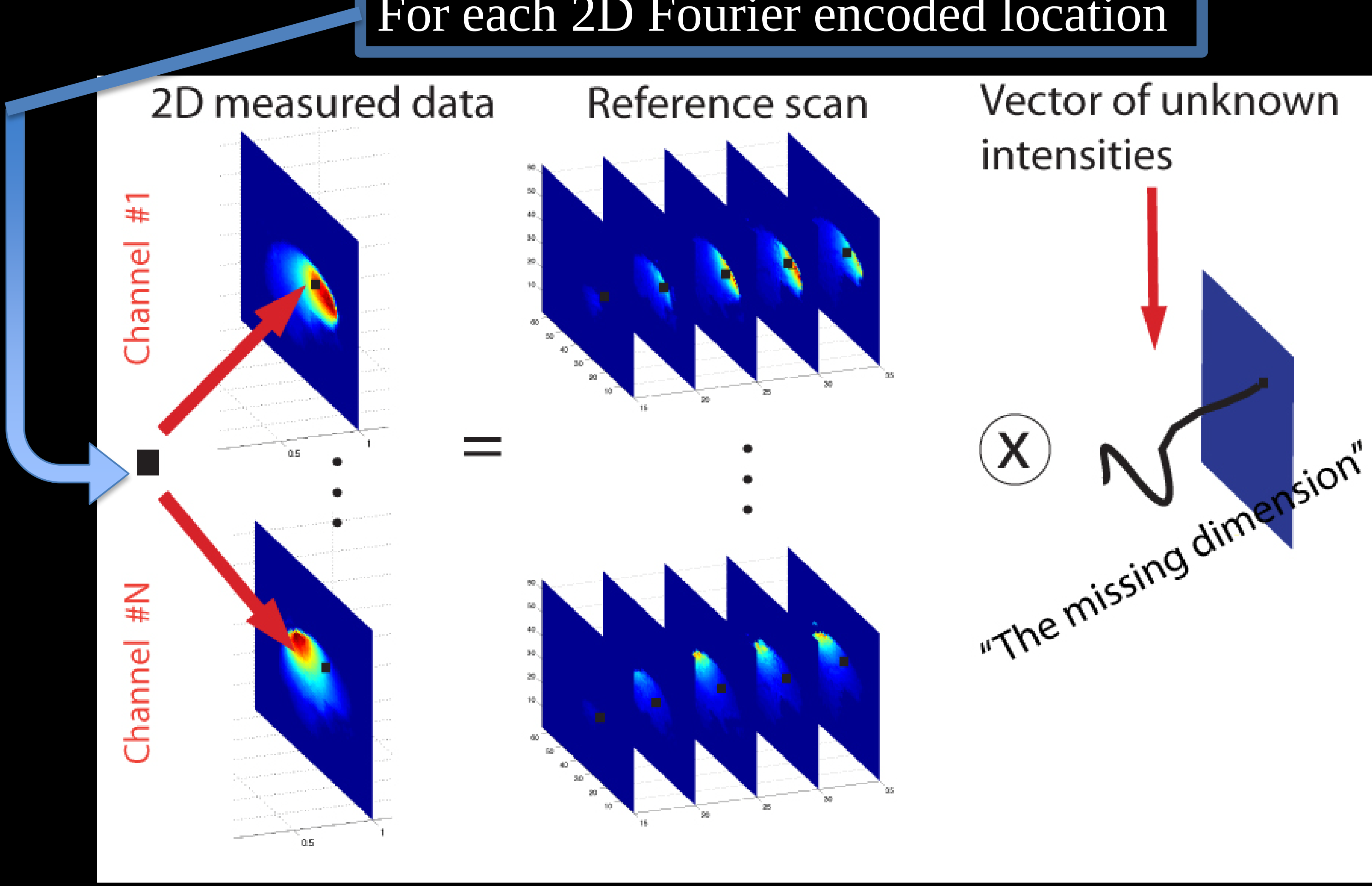
Reference scan / Forward model

- Full Fourier encoded volumetric (3D) scan
- Comprises in-vivo coil sensitivity information



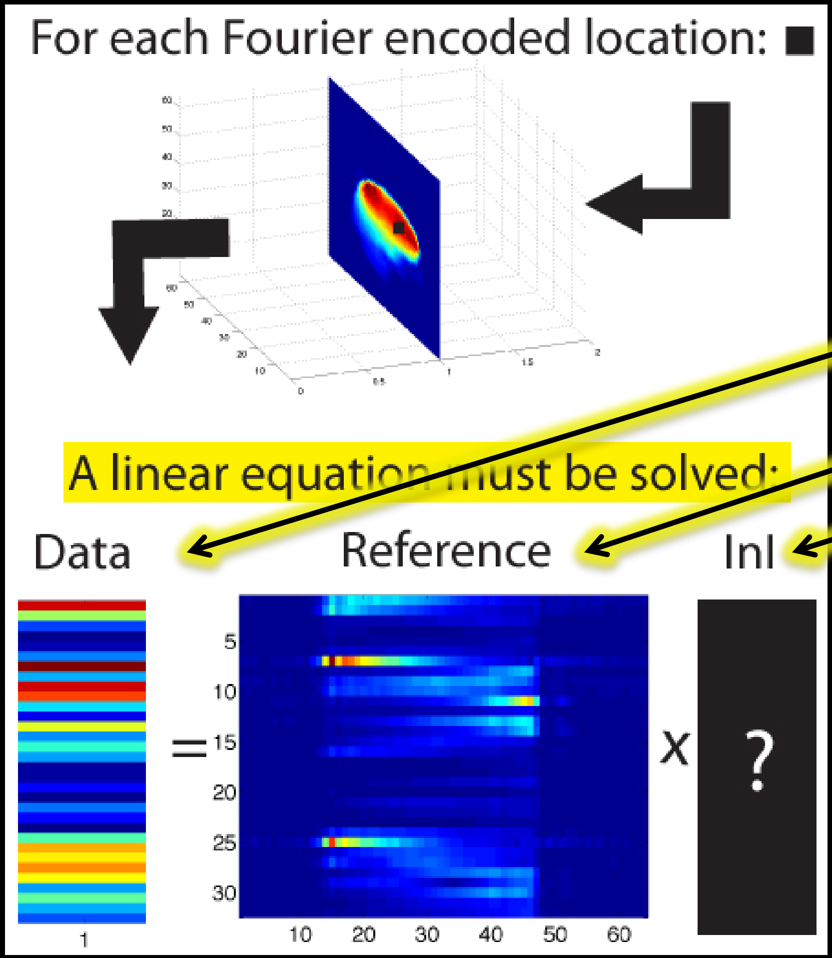
Inverse reconstruction

For each 2D Fourier encoded location



Methods

Linear inverse problem



The linear model is:

$$\mathbf{Y}(t) = \mathbf{A}\mathbf{X}(t)$$

$\mathbf{X}(t)$ = vector of unknown contrast values

$\mathbf{A}$ = matrix of reference intensity values

$\mathbf{Y}(t)$ = vector of channel measurements

t=time

Inverse estimation

- The ordinary least squares estimate:

$$\mathbf{X}_{LS}(t) = (\mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'\mathbf{Y}(t)$$

  – Ill-posed: $(\mathbf{A}'\mathbf{A})^{-1}$ is (close to) singular.
- Measurements are corrupted by noise.
  – Noise assumed Gaussian with covariance Σ_n.
- Min Norm Estimation (MNE) made well posed:
  – Include a regularizing term to LS-solution

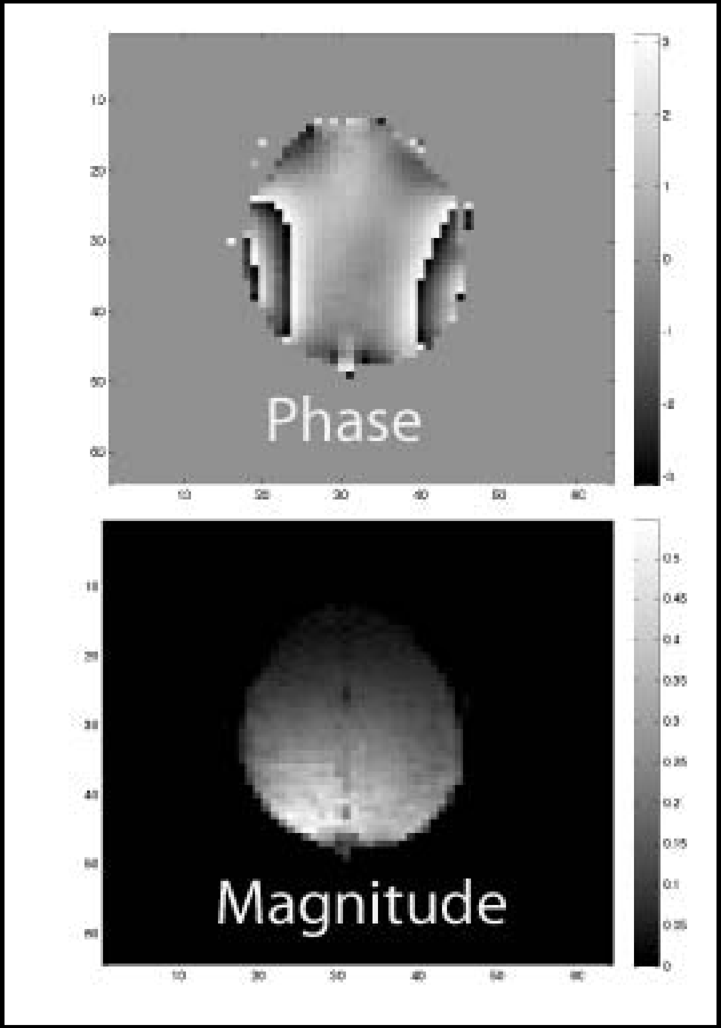
$$\mathbf{X}(t)_{MNE} = (\mathbf{A}'\Sigma_n^{-1}\mathbf{A} + \gamma^{-1}\mathbf{1})^{-1}\mathbf{A}'\Sigma_n^{-1}\mathbf{Y}(t)$$

Phase constraint

- Reference scan:
  - phase information

$$\begin{bmatrix} \mathrm{Re}(\mathbf{Y}) \\ \mathrm{Im}(\mathbf{Y}) \end{bmatrix} = \begin{bmatrix} \mathrm{Re}(\mathbf{A}) \\ \mathrm{Im}(\mathbf{A}) \end{bmatrix} [\mathbf{X}]$$

Constraint: $\mathbf{X}$ is real



Statistical form of inverse problem

- Statistical interpretation of the regularization
  - Gaussian prior distribution for $\mathbf{X}(t)$.
- Regularization is the variance of this Gaussian:
  - Quantifies solution's *a priori* "overall power".
- Bayes:

Posterior ↓ — Likelihood (linear model) ↓ — Prior (Gaussian) ↓

$$p(\mathbf{X}(t)\,|\,\mathbf{Y}(t),\gamma) \propto p(\mathbf{Y}(t)\,|\,\mathbf{X}(t))\,p(\mathbf{X}(t)\,|\,\gamma)$$

Statistical interpretation of MNE

- With a fixed $\gamma = \hat{\gamma}$
  - MNE is maximum a Posteriori Estimate (MAP):

$$\mathbf{X}_{MAP}(t) = \arg\max_{\mathbf{X}} p(\mathbf{X}(t) \mid \mathbf{Y}(t), \hat{\gamma})$$

$$\mathbf{X}_{MAP}(t) = \mathbf{X}_{MNE}(t) = (\mathbf{A}'\Sigma_n^{-1}\mathbf{A} + \hat{\gamma}^{-1}\mathbf{1})^{-1}\mathbf{A}'\Sigma_n^{-1}\mathbf{Y}(t)$$

Aspects of regularization

- The regularization reflects the (prior) signal variance:
  - This means it is also connected to SNR.
  - If we know noise covariance, the regularization parameter can be estimated (it is identifiable).
- Regularization should / could depend on time:
  - Variance-type parameter -> better with more data.
  - Dynamical estimation across time.

Time independent regularization

- Gaussian noise & Gaussian prior:
  - Joint distribution of $\mathbf{Y}(t), \mathbf{X}(t)$ is Gaussian
- We can integrate over $\mathbf{X}(t)$ analytically:

$$p(\mathbf{Y}(t) \mid \gamma) = \int p(\mathbf{Y}(t), \mathbf{X}(t) \mid \gamma) \mathrm{d}\mathbf{X}(t)$$

$$\hat{\gamma} = \arg\max \prod_{t=1}^{N_t} p(\mathbf{Y}(t) \mid \gamma)$$

- Type-II Maximum Likelihood (ML-II) estimate (optimization *e.g.*, with “fmincon”).

Time-dependent regularization

- The estimate $\gamma(t-1)$ can be used as a “prior” for $\gamma(t)$ to stabilize time-dependent estimation.

$$\hat{\gamma}(t) = \arg\max p(\mathbf{Y}(t) \mid \gamma(t))\, p(\gamma(t) \mid \gamma(t-1));$$

- Simplest form:

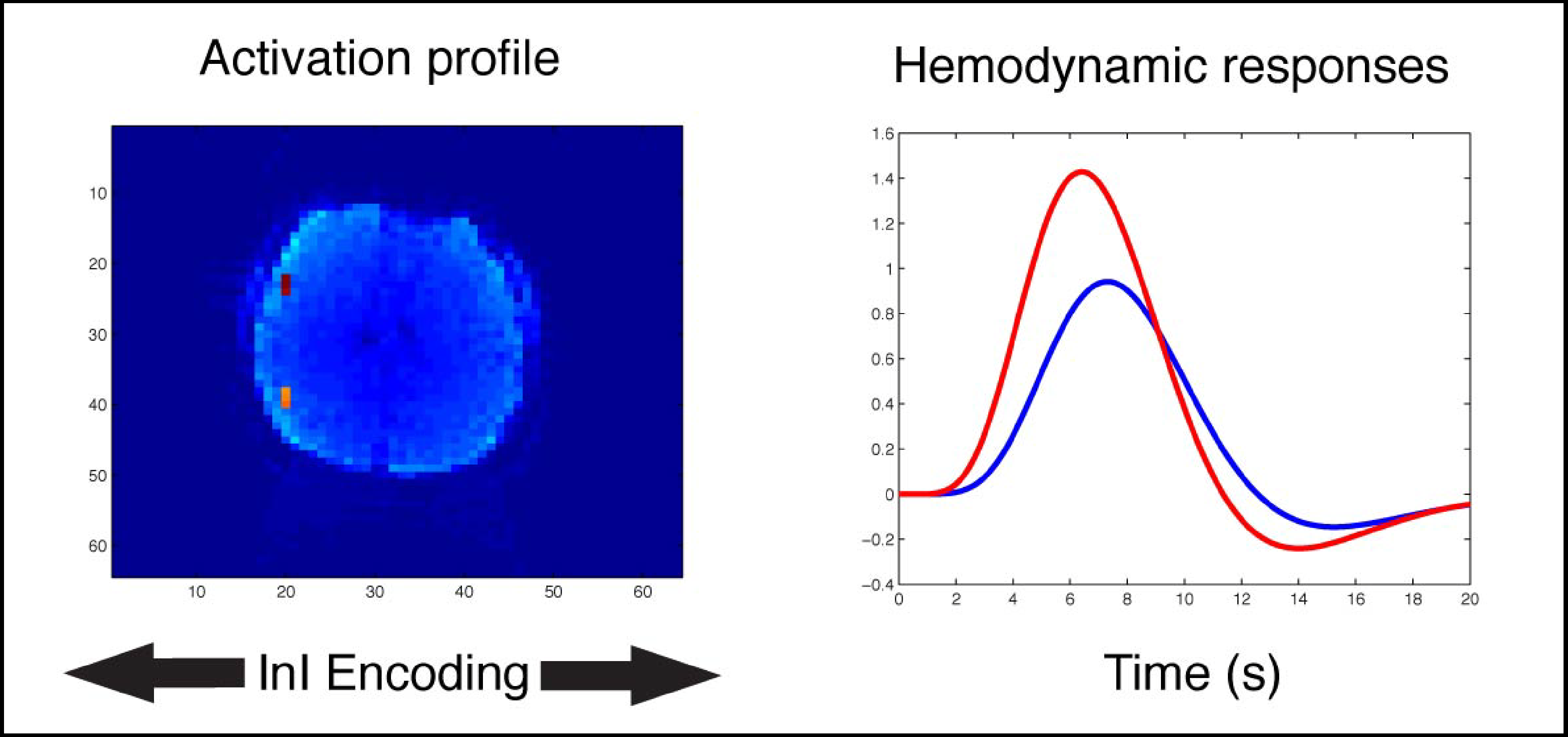
$$p(\gamma(t) \mid \gamma(t-1)) = \text{uniform}(\hat{\gamma}(t-1) \pm \Delta)$$

- The Δ determines the temporal smoothness.

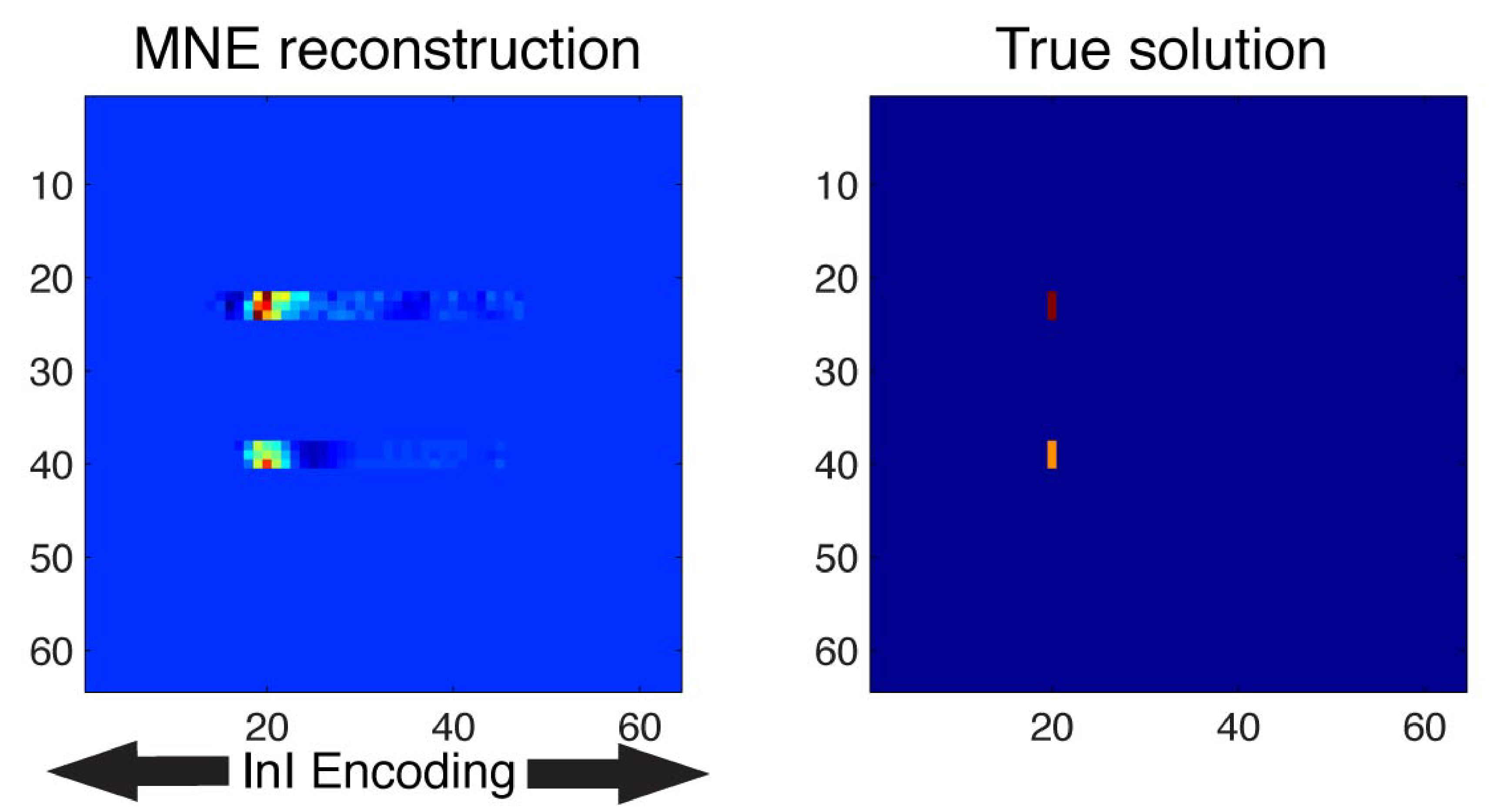
Results

Simulated data

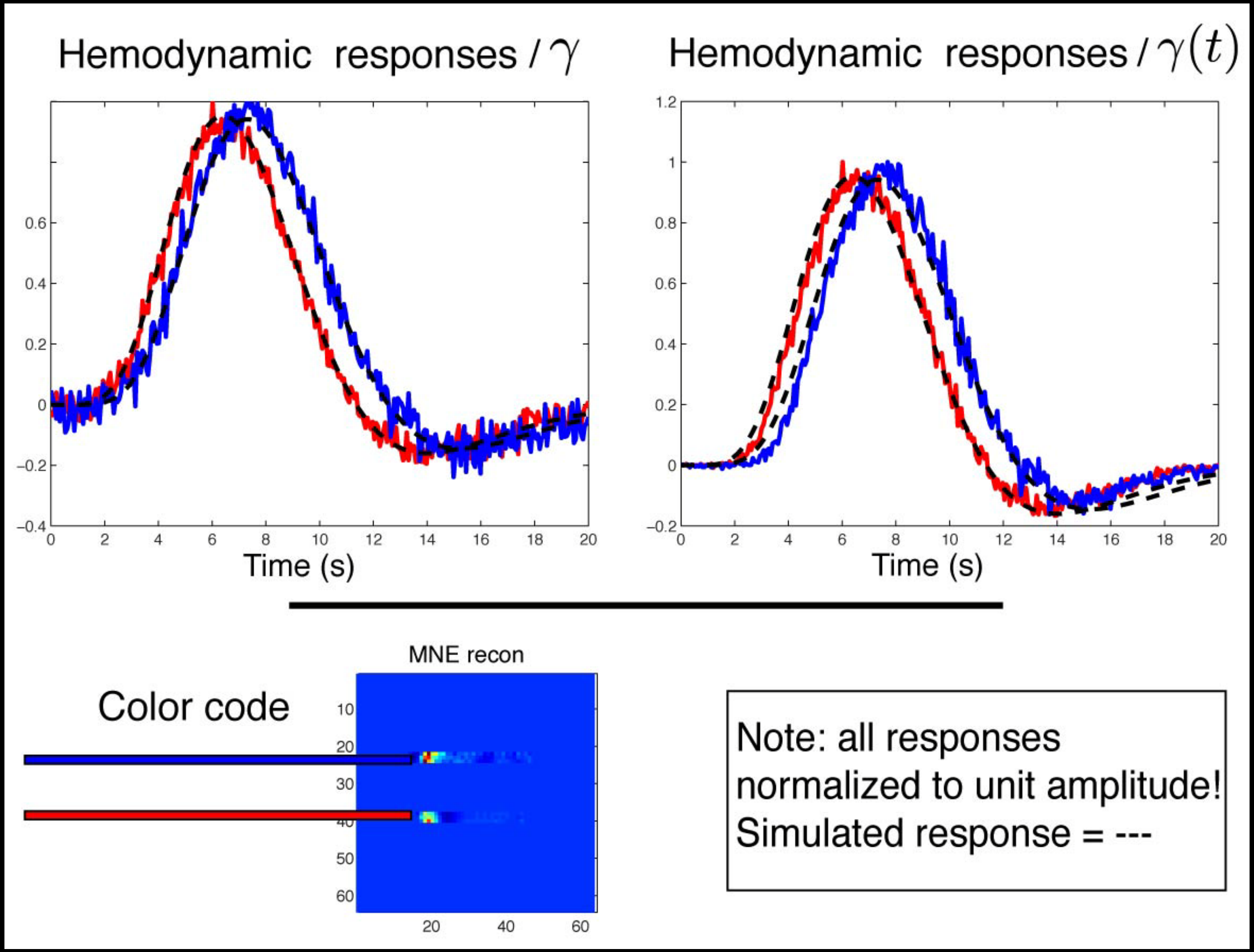
- Two “activation clusters” (on top of the a reference slice).
- Identical noise added -> different overall SNR.



The MNE recon



MNE reconstructed HDRs

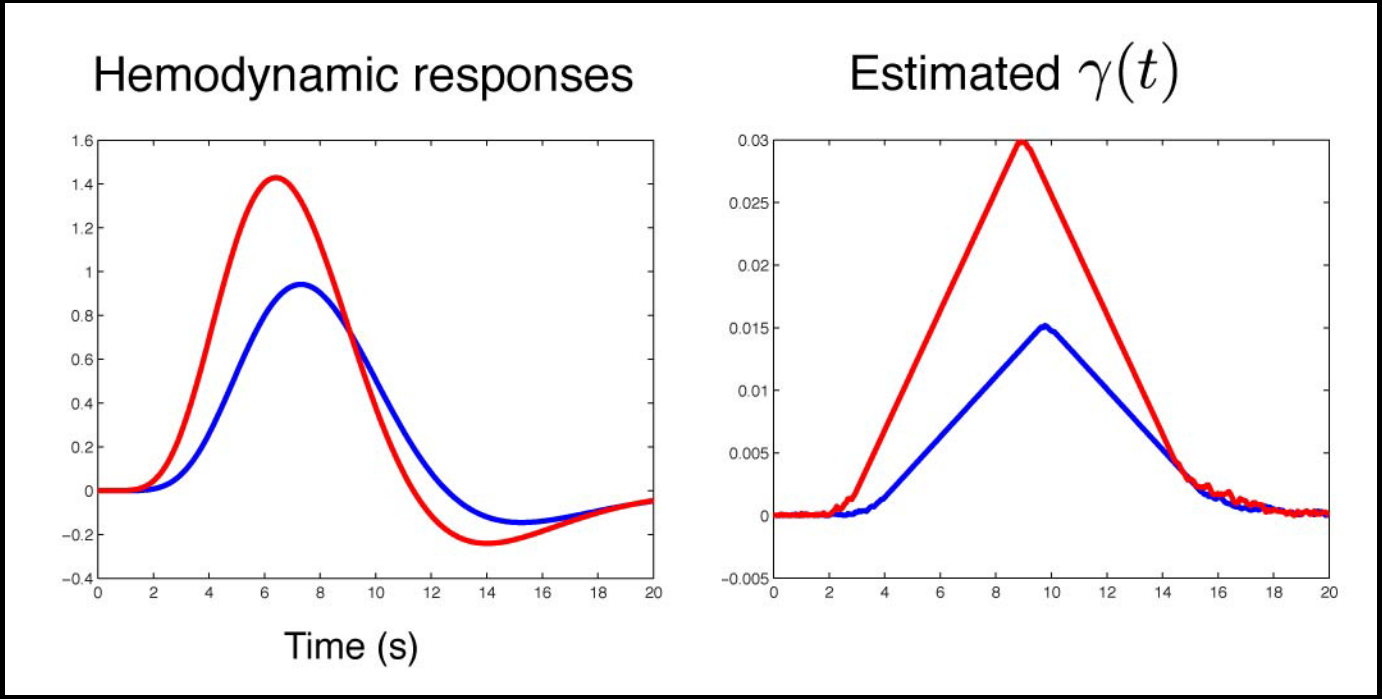


The time-dependent regularization "shrinks" the estimates towards zero when signal is below noise.

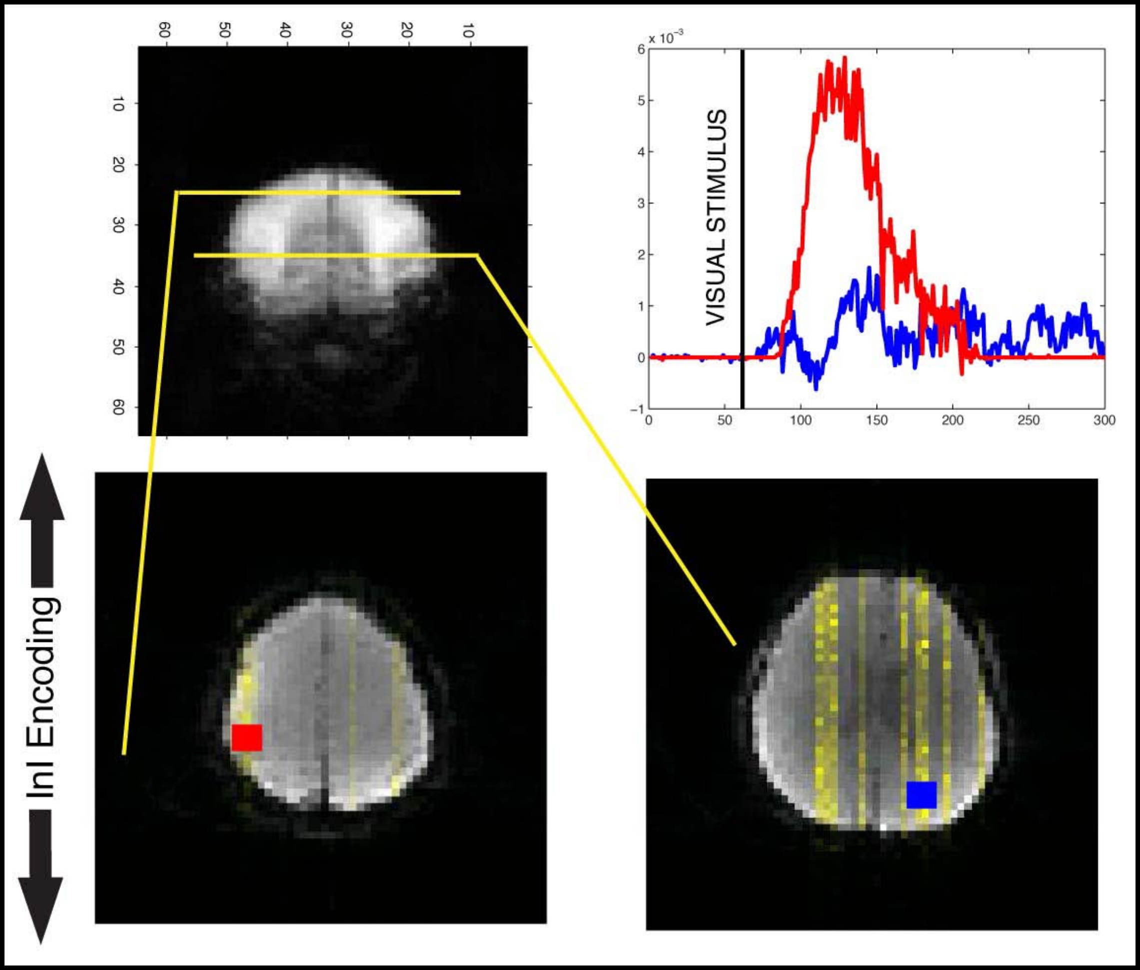
Some lag in the estimates, but different response offsets are more clearly visible.

The time dependence of $\hat{\gamma}(t)$

- The piecewise regularization time course results from "uniform prior" dynamics.
- Relative activation magnitude and timing are clearly visible!



Empirical data (visually cued motor response)



The proposed method leads to enhanced estimation of onset latencies and temporal response characteristics on single-subject level.

Group analysis is required to get sufficient SNR and suppress variability of vascular responses with basic MNE-InI (Fa-Hsuan Lin *et al.*, Submitted, 2010).

Summary

Time-dependent regularization

- The SNR in the hemodynamic response may/will vary in time.
  - This can be incorporated into the regularization.
  - Dynamic prior model is required for stability.

- The proposed method can enhance estimation of onset latencies & temporal characteristics.
  - Uncertainty on the HDR is taken more accurately into account in the inverse solution.