

Random Effects vs Fixed Effects Group Variance Models

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MEG Users Talk

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Outline

- Inference primer
- Random Effects (RFX) - Unweighted
- Fixed Effects (FFX)
- Inference
- Conclusion
- Weighted Random Effects
- Mixed Effects

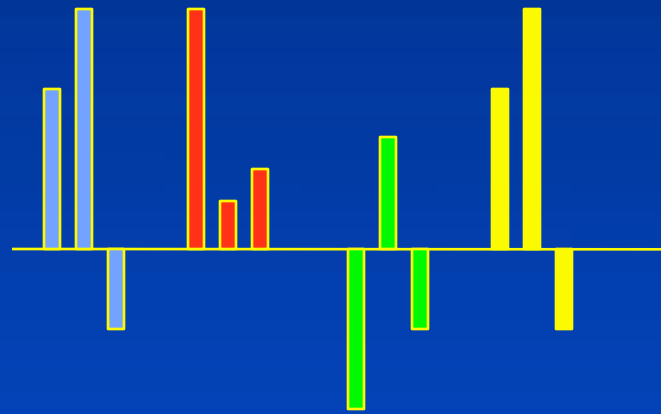
Inference 101

$$t_{\text{dof}} = \frac{\text{mean}}{\text{SE}}$$

- Mean – bigger is better
- SE (Standard Error) – small is better
- DOF – bigger is better

Typical Data

Multiple
Individual
Observations



N=4 subjects

M=3 observations (presentation)

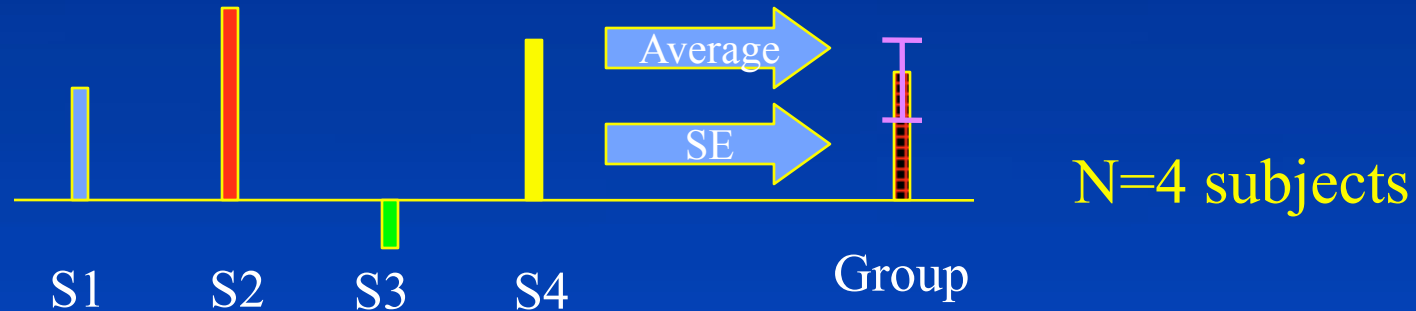
Individual
Means and
Variances



Now What?

RFx (Unweighted)

$$t = \frac{\text{mean}}{\text{dof} \cdot \text{SE}}$$



Throw away information about within-subject variation.
Ie, pretend within-subject variation = 0.

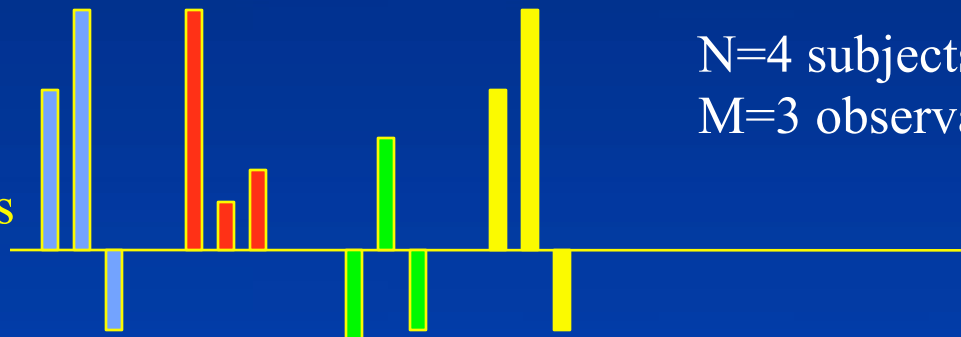
- $\text{DOF} = N - 1 = 3$
- Group average is mean of individual means.
- Group variance is variance across individual means.
- RFx variance is a measure of population variance.
- $\text{SE} = \sqrt{\text{var}/N}$

FFx

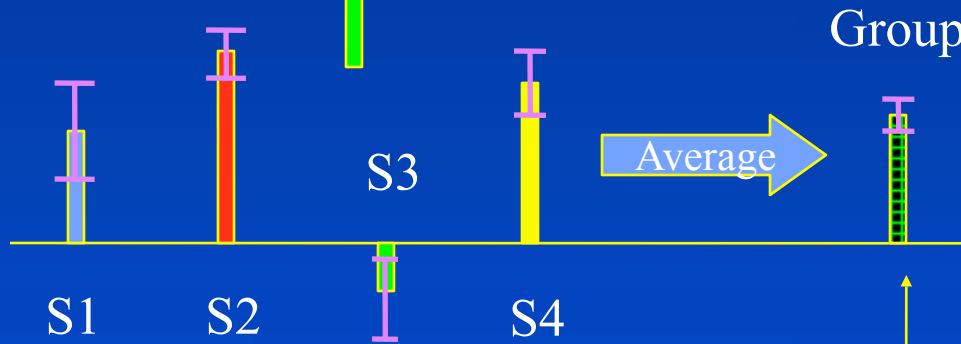
$$t = \frac{\text{mean}}{\text{dof SE}}$$

N=4 subjects
M=3 observations

Multiple
Individual
Observations



Individual
Means



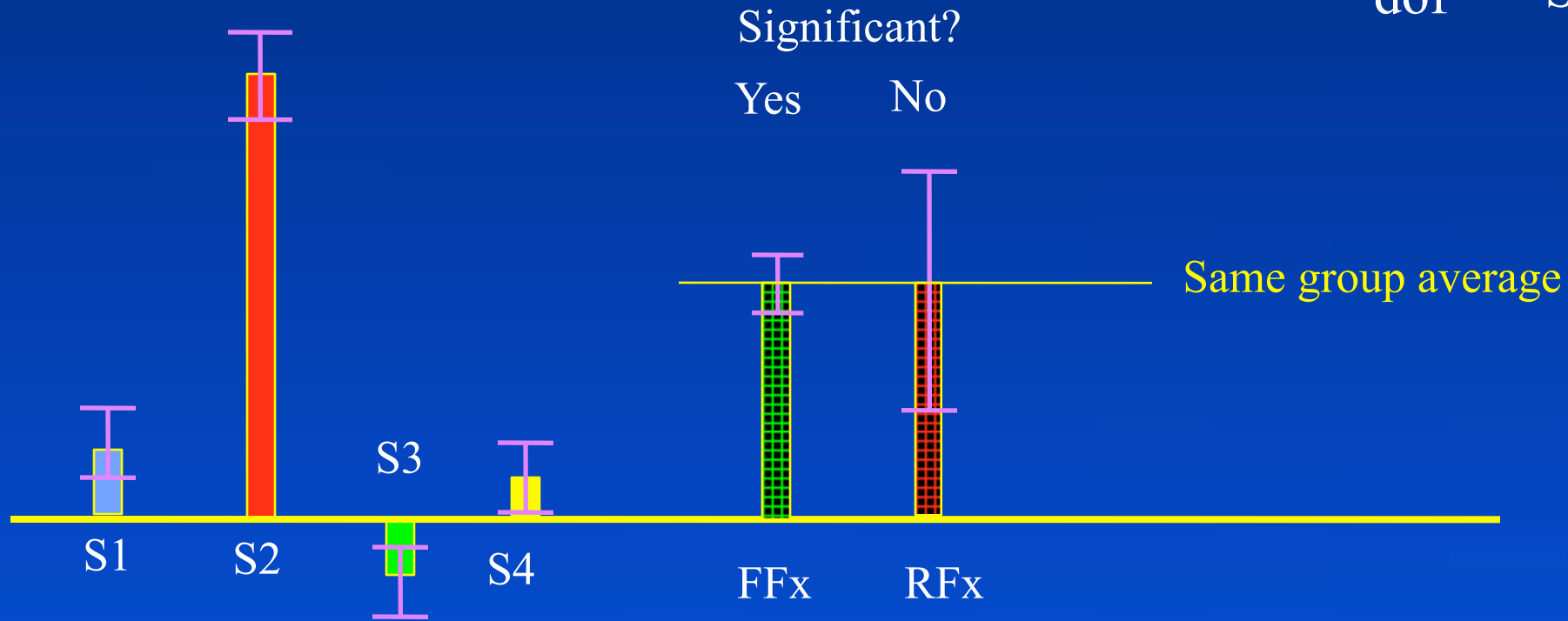
Individual
Variances



- $DOF = N*(M-1)=8$
- Group Mean = Mean of Individuals (same as RFX)
- Variance is measure of observation noise (eg, white)
- $SE = \text{sqrt}(\text{mean var}/(N*M))$
- Similar to treating all observations as if they came from the same subject.

FFx vs RFx Inference

$$t = \frac{\text{mean}}{\text{SE}}$$



- How likely am I to see this effect in one or more of my subjects (ie, a cohort)? Answered by FFX.
- How likely am I to see this effect in the general population? Answered by RFX.

RFx vs FFx

$$t = \frac{\text{mean}}{\text{SE}}$$

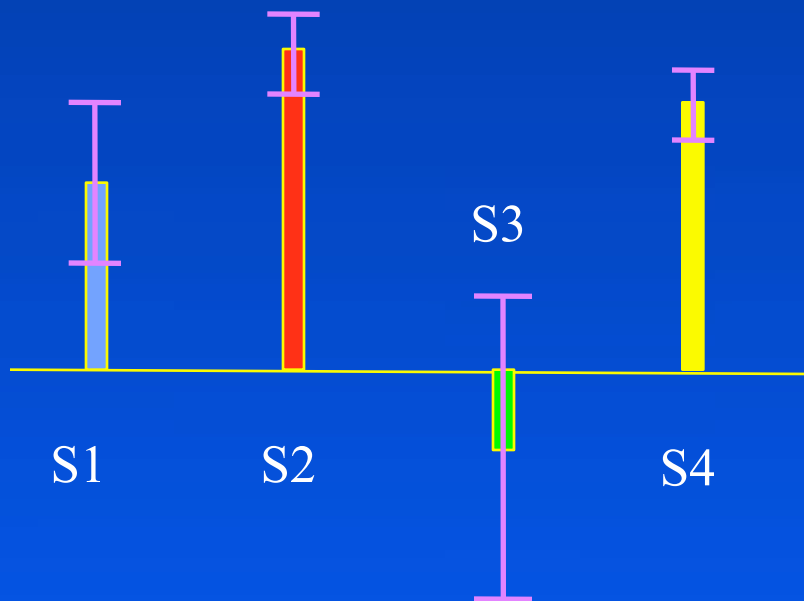
dof

| | RFx | FFx |
|-----------|------------------------------|------------------------------|
| Mean | Same | Same |
| DOF | #Subj (~20) | #Subj*#Obs (~1000) |
| Power | Weak | Strong |
| Variance | Population (between-subj) | Observation (within-subj) |
| Inference | Population | Cohort |

RFx - Weighted

- Multiple observations per subject
- Unequal variances

$$\text{Average} = \text{Sum}(W_i * S_i)$$
$$\text{Var} = \text{Sum}(W_i * (S_i - \text{Average}))$$



$$\text{Weight: } W_i = 1/\text{StdDev}_i$$
$$\text{Normalized: } \text{Sum}(\text{Weights}) = 1$$

- Same as unweighted when variances are equal
- Same trade-offs as unweighted

Mixed Effects

- Variance computed by RE_{FX} is a mixture of within-subject and between-subject variances.
- Use individual variances to help tease apart the within-subject from the between-subject
- Nonlinear
- Fit with Maximum-Likelihood/Bayes