Fast Regularized Reconstruction Tools for QSM and DSI

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L2-Regularized Reconstruction

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- Matlab tools that achieve dramatic speed-up relative to iterative algorithms will be presented

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- Matlab tools that achieve dramatic speed-up relative to iterative algorithms will be presented
- Two representative applications are considered,
  - Quantitative Susceptibility Mapping (QSM)
  - Diffusion Spectrum Imaging (DSI)

Quantitative Susceptibility Mapping (QSM)

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- Susceptibility correlates well with tissue iron concentration, especially in iron rich deep gray matter structures [1,2]

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- Susceptibility mapping requires the solution of an inverse problem,

$$\mathbf{F}^H \mathbf{D} \mathbf{F} \, \chi = \phi$$

$\mathbf{D}$: diagonal matrix; $\mathbf{F}$: DFT; χ: unknown susceptibility; ϕ: unwrapped phase

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χ: to be estimated

ϕ: measured

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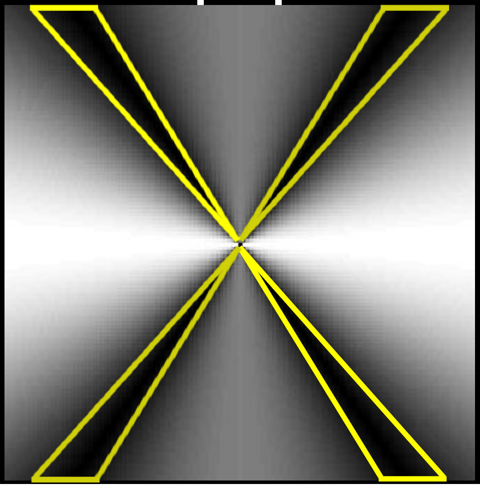
$$\mathbf{F}^H \mathbf{D} \mathbf{F} \; \chi = \phi$$

$$\mathbf{D} = \frac{1}{3} - \frac{{k_z}^2}{k^2}$$

⇒ Undersamples k-space on a conical surface

|D|



Regularized QSM

- Solution of inverse problem is facilitated by regularization that imposes prior knowledge [1]

$$\chi = argmin_{\chi} \underbrace{\|\boldsymbol{\phi} - \mathbf{F}^{H}\mathbf{D}\mathbf{F}\boldsymbol{\chi}\|_2^{\,2}}_{\text{data consistency}} + \lambda \cdot \underbrace{\|\mathbf{G}\boldsymbol{\chi}\|_2^{\,2}}_{\ell_2 \text{ over gradients}}$$

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$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_x \\ \mathbf{G}_y \\ \mathbf{G}_z \end{bmatrix}$$

gradient in 3D

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- Prior: underlying susceptibility map is smooth

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- Solution can be evaluated in closed-form

$$\boldsymbol{\chi} = (\mathbf{F}^H \mathbf{D}^2 \mathbf{F} + \lambda \cdot \mathbf{G}^H \mathbf{G})^{-1} \mathbf{F}^H \mathbf{D} \mathbf{F} \boldsymbol{\phi}$$

- The minimizer can be computed efficiently given that the matrix inversion is rapidly performed

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Fast Regularized QSM

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- Gradient in image space can be represented in k-space by multiplication with a diagonal matrix $\mathbf{E}$

$$\mathbf{G} = \mathbf{F}^H \mathbf{E} \, \mathbf{F}$$ where $\mathbf{E}(i,i) = 1 - e^{(-2\pi\sqrt{-1}k(i,i)/N)}$

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- $\mathbf{E}$ is simply the k-space representation of the difference operator $\delta_i - \delta_{i-1}$

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- With this formulation, closed-form solution becomes

$$\chi = \mathbf{F}^H \mathbf{D}[\mathbf{D}^2 + \lambda \cdot (\mathbf{E}_x^2 + \mathbf{E}_y^2 + \mathbf{E}_z^2)]^{-1} \mathbf{F} \boldsymbol{\phi}$$

all matrices diagonal

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- **Total cost:** Two FFTs and multiplication of diagonal matrices

Regularized QSM Results

- Numerical Phantom
  - Three compartments (gray, white, CSF) with constant χ
  - Phase ϕ computed from true χ, and peak-SNR = 100 noise added
  - Regularization parameter λ chosen to minimize RMSE in reconstructed χ

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- <u>In Vivo 3D SPGR</u>
  - Healthy subject at 1.5T with resolution $0.94\times0.94\times2.5\text{mm}^3$
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- Comparison of methods
  i. Iterative solution using Nonlinear Conjugate Gradient [1,2]
  ii. Proposed closed-form solution

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  - i. Iterative solution → converges to closed-form solution
  - ii. Proposed closed-form solution

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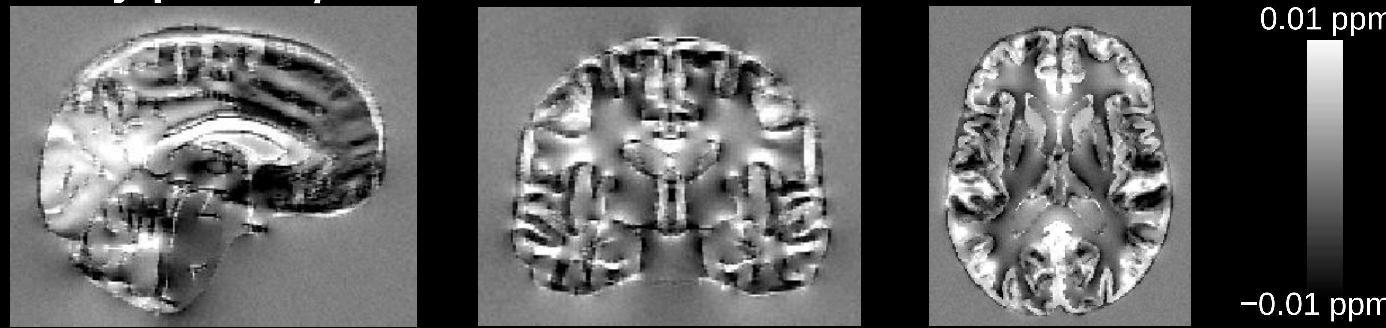
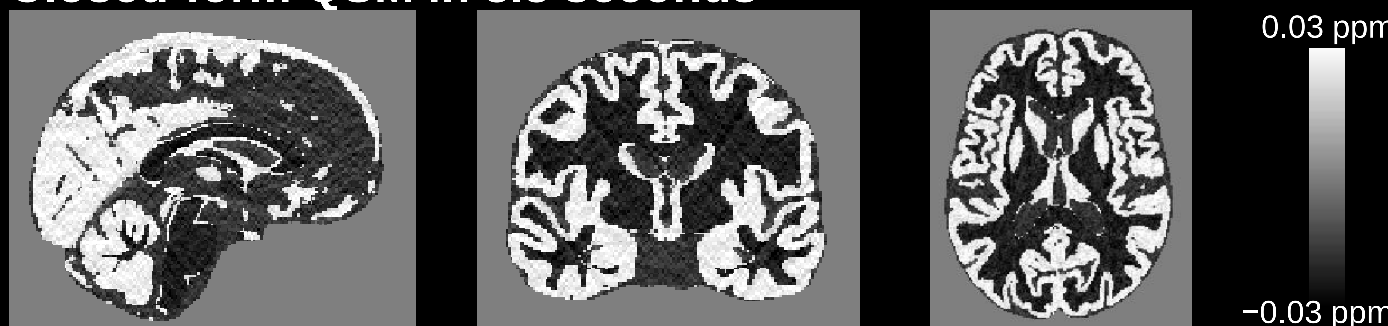
Numerical Phantom

Noisy phase ϕ

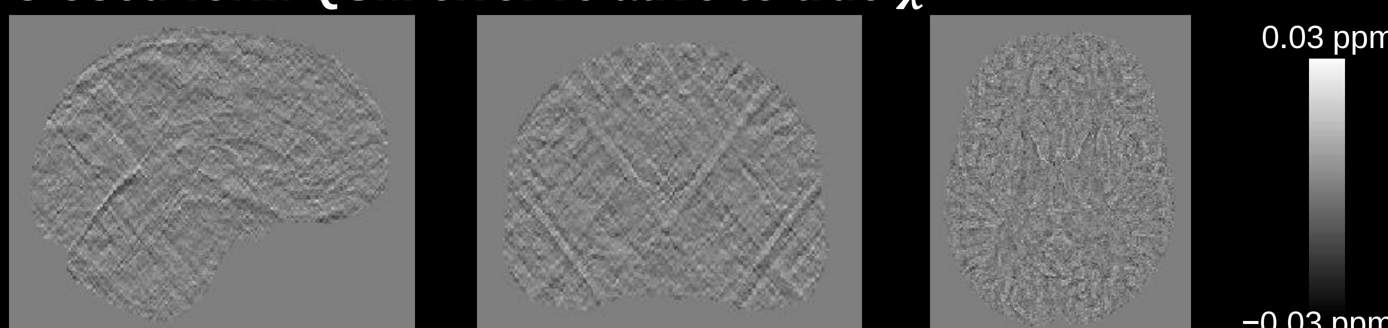
error due to noise: 5.9% RMSE



Closed-form QSM in 3.3 seconds



Closed-form QSM error relative to true χ



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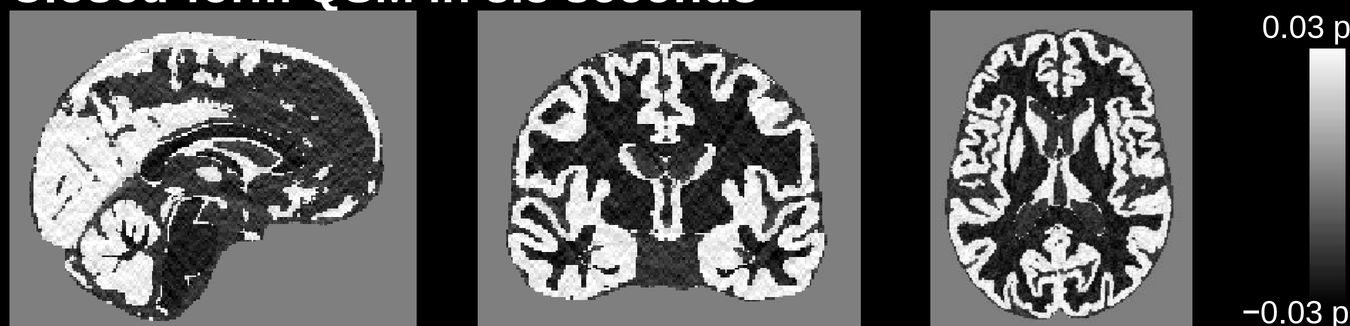

QSM Method	Recon Time	Error relative to true χ
Closed-form	**3.3 seconds**	17.4% RMSE
Iterative [1,2], 100 iters	65 minutes	18.0% RMSE

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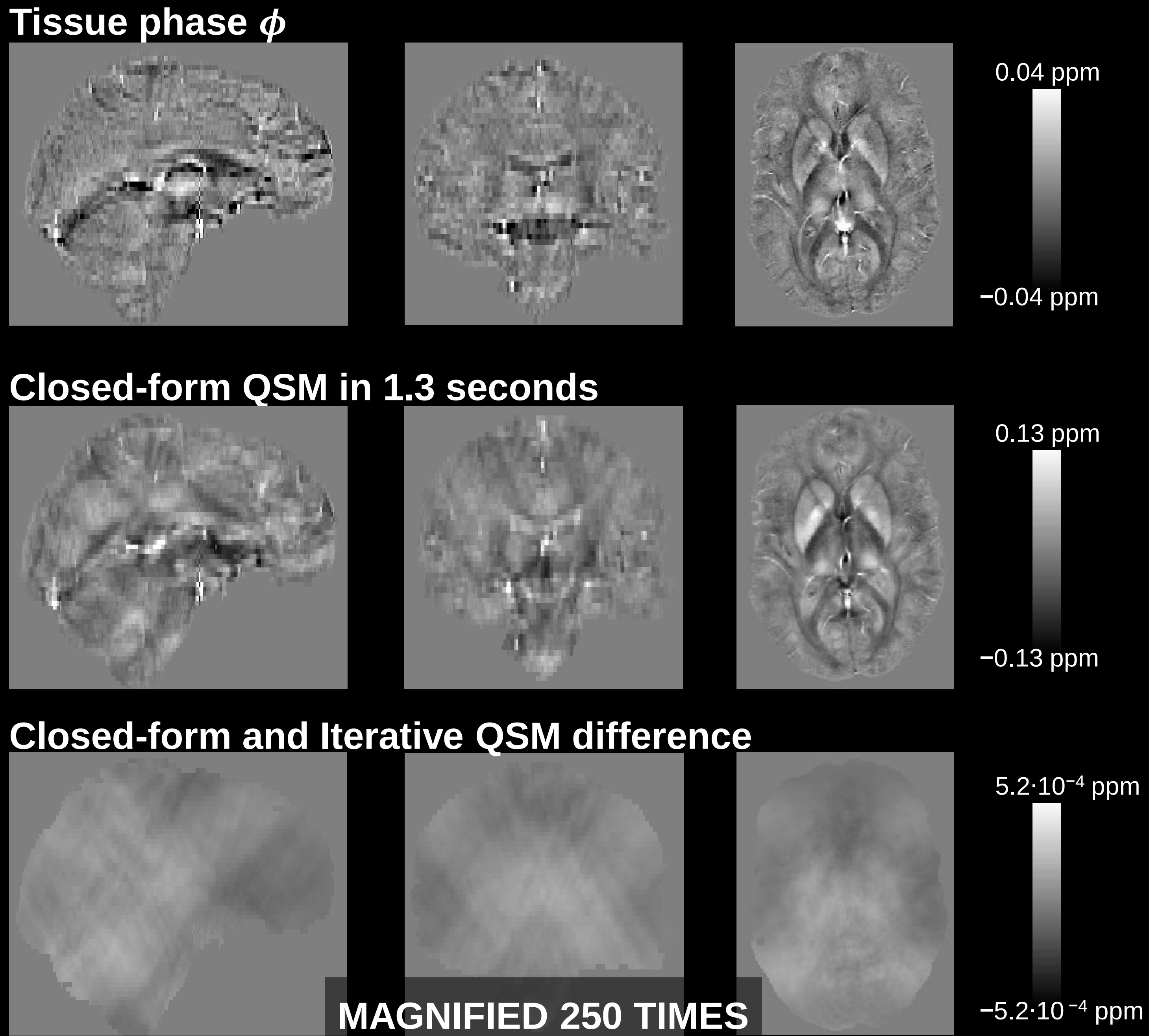
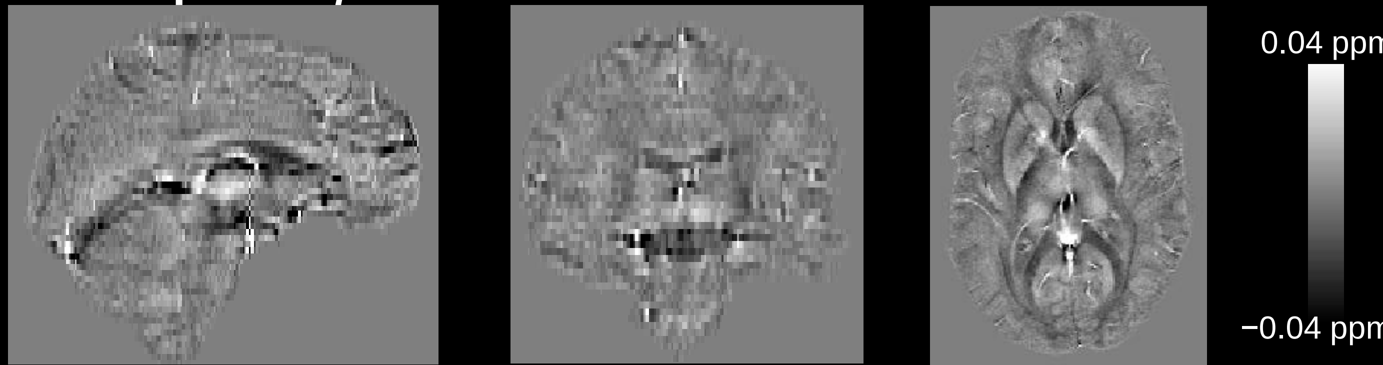
In Vivo QSM



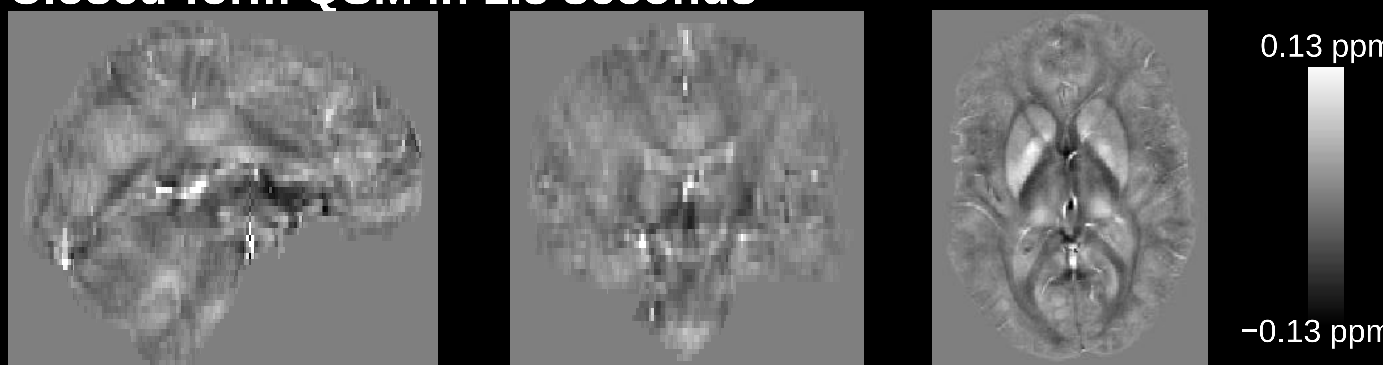
Tissue phase ϕ
0.04 ppm
−0.04 ppm
Closed-form QSM in 1.3 seconds
0.13 ppm
−0.13 ppm
Closed-form and Iterative QSM difference
5.2·10^{-4} ppm
−5.2·10^{-4} ppm
MAGNIFIED 250 TIMES

In Vivo QSM

Tissue phase ϕ



Closed-form QSM in 1.3 seconds



QSM Method	Recon Time
Closed-form	**1.3 seconds**
Iterative Conj Grad [1,2], 100 iters	29 minutes

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Diffusion Spectrum Imaging (DSI)

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- And reveals complex distributions of fiber orientations
- DSI requires full sampling of q-space (DTI needs ≥7 points)

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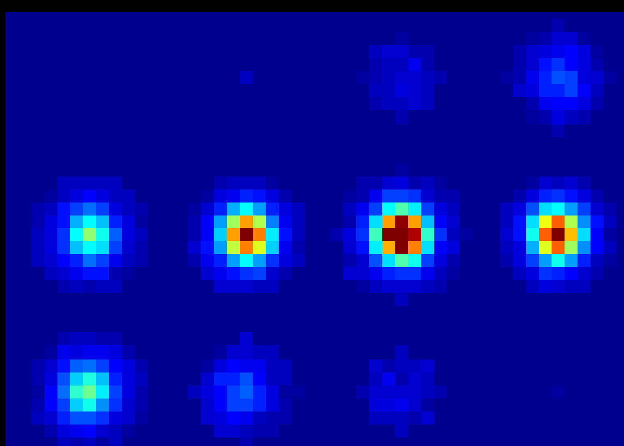
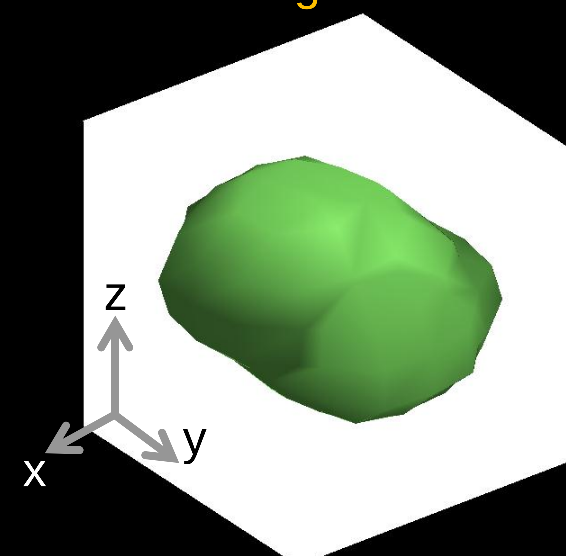
Q-space of a single voxel
515 directions



DFT

Probability Density Function (pdf)
of a single voxel

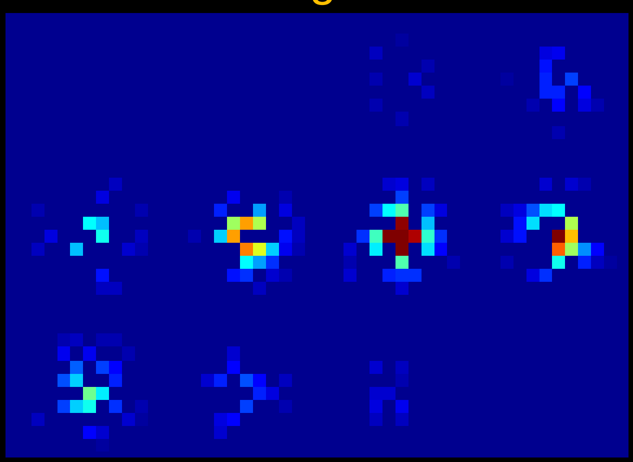


Sampling full q-space takes ~1 hour

Undersampled DSI

- To reduce scan time, undersample q-space
- Use sparsity prior to recon the pdfs via Compressed Sensing

Undersampled q-space
of a single voxel



Compressed

Sensing

Probability Density Function (pdf)
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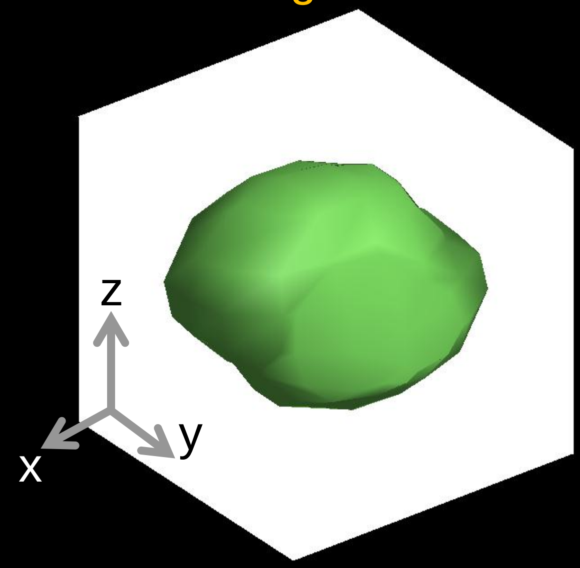

Undersampled DSI

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i. Wavelet + Total Variation [1]

$$min_{\boldsymbol{p}} \|\mathbf{F}_{\Omega}\boldsymbol{p} - \boldsymbol{q}\|_2{}^2 + \alpha \cdot \|\boldsymbol{\Psi}\boldsymbol{p}\|_1 + \beta \cdot \mathrm{TV}(\boldsymbol{p})$$

undersampled DFT ($\mathbf{F}_{\Omega}$), pdf ($\boldsymbol{p}$), q-samples ($\boldsymbol{q}$), wavelet ($\boldsymbol{\Psi}$), total variation (TV)

1. Menzel *et al* MRM 2011

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ii. Dictionary-FOCUSS [2]

- ❑ Create a dictionary $\mathbf{D}$ from a training dataset of pdfs using K-SVD algorithm [3] → tailored for sparse representation
- ❑ Impose sparsity constraint via FOCUSS algorithm [4] by solving

$$min\|\boldsymbol{x}\|_1 \quad \text{such that} \quad \mathbf{F}_{\Omega}\mathbf{D}\boldsymbol{x} = \boldsymbol{q}$$

1. Menzel *et al* MRM 2011
2. Bilgic *et al* MRM 2012
3. Aharon *et al* IEEE TSP 2006
4. Gorodnitsky *et al* IEEE TSP 1997

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↘ Dictionary transform coefficients

Iterative DSI Reconstruction

- Dictionary-FOCUSS [1] yields up to 2-times RMSE reduction using compared to Wavelet+TV

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- Full-brain recon for 10^5 voxels: ~ 10 DAYS of computation

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Iterative DSI Reconstruction

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- Compressed Sensing recon is iterative, with processing times > 10 sec / voxel for both methods
- Full-brain recon for 10^5 voxels: ~ 10 DAYS of computation
- Two L2-based methods that are 1000-fold faster with image quality similar to Dictionary-FOCUSS are proposed:
  i. Tikhonov regularization
  ii. Principal Component Analysis (PCA)

1. Bilgic *et al* MRM 2012

Tikhonov Regularization

- Dictionary-FOCUSS iteratively solves

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- Instead, consider

$$min\ \|\mathbf{F}_\Omega \mathbf{D}\boldsymbol{x} - \boldsymbol{q}\|_2{}^2 + \lambda \cdot \|\boldsymbol{x}\|_{\mathbf{2}}{}^{\mathbf{2}}$$

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- Solution: $\widetilde{\boldsymbol{x}} = ((\mathbf{F}_\Omega \mathbf{D})^H \mathbf{F}_\Omega \mathbf{D} + \lambda \mathbf{I})^{-1} (\mathbf{F}_\Omega \mathbf{D})^H q$

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Singular Value Decomposition: $\mathbf{F}_\Omega \mathbf{D} = \mathbf{U\Sigma}\boldsymbol{V}^H$

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$$\mathbf{F}_\Omega \mathbf{D} = \mathbf{U}\boldsymbol{\Sigma} V^H$$

$$\boldsymbol{\Sigma}^+ = (\boldsymbol{\Sigma}^H \boldsymbol{\Sigma} + \lambda \mathbf{I})^{-1} \boldsymbol{\Sigma}^H$$

$$\widetilde{\boldsymbol{x}} = \underbrace{\mathbf{V}\boldsymbol{\Sigma}^+ \mathbf{U}^H}_{\text{compute once}} q$$

PCA Reconstruction

- PCA: approximates data points using a linear combo of them to retain the maximum variance in the dataset

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$$\mathbf{Z}\mathbf{Z}^H = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^H$$

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- Pick the first T columns of $\mathbf{Q}$ corresponding to largest eigvals: $\mathbf{Q}_{T}$

$$\boldsymbol{pca} = \mathbf{Q}_{T}^{H}(\boldsymbol{p} - \boldsymbol{p}_{mean})$$

↓

T - dimensional
pca coefficients

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$$\boldsymbol{pca} = \mathbf{Q}_T^H(\boldsymbol{p} - \boldsymbol{p}_{mean})$$

- The location of $\boldsymbol{pca}$ in the pdf space,

$$\boldsymbol{p}_T = \mathbf{Q}_T\boldsymbol{pca} + \boldsymbol{p}_{mean}$$

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- Least-squares approximation in T - dimensions,

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- In PCA coordinates,

$$min_{\boldsymbol{pca}} \, \|\mathbf{F}_{\Omega}\mathbf{Q}_{T}\boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_{\Omega}\boldsymbol{p}_{mean})\|_2^2$$

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$$min \, \|\mathbf{F}_{\Omega}\boldsymbol{p}_{\boldsymbol{T}} - \boldsymbol{q}\|_2{}^2$$

- In PCA coordinates,

$$min_{\boldsymbol{pca}} \, \|\mathbf{F}_{\Omega}\mathbf{Q}_T\boldsymbol{pca} - (\boldsymbol{q} - \mathbf{F}_{\Omega}\boldsymbol{p}_{mean})\|_2^2$$

- Closed-form solution:

$$\widetilde{\boldsymbol{pca}} = \underbrace{\mathrm{pinv}(\mathbf{F}_{\Omega}\mathbf{Q}_T)}_{\text{compute once}}(\boldsymbol{q} - \mathbf{F}_{\Omega}\boldsymbol{p}_{mean})$$

DSI Acquisition

- 2.3 mm isotropic with b_{max} = 8000 s/mm^2 at 3T
- Connectom gradients and 64-chan head coil [1]
- 515 q-space points collected in 50 min

1. Keil *et al* MRM 2012

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- Recon experiments at accelerations R = 3, 5 and 9

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- Comparison of methods:
  - i. Wavelet + TV [2]
  - ii. Dictionary-FOCUSS [3]
  - iii. Tikhonov regularization
  - iv. PCA

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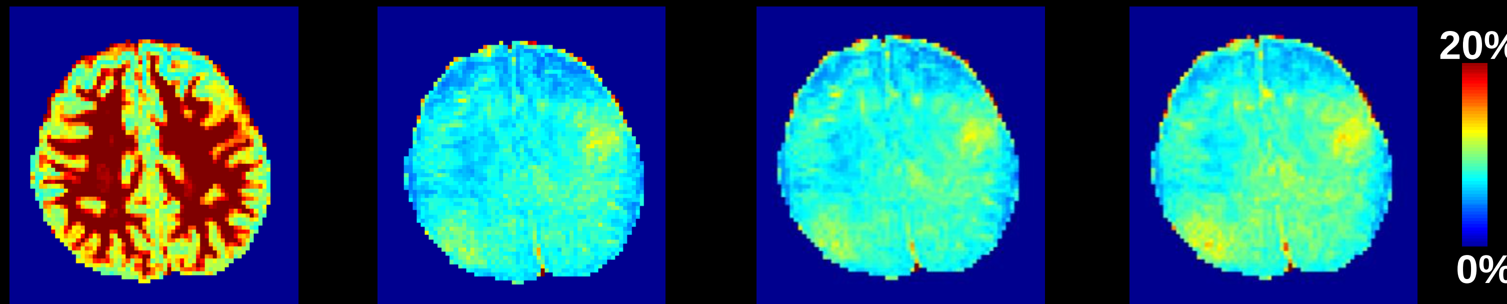
pdf reconstruction error maps

Recon Time	1190 min	530 min	0.6 min	0.4 min
	Wavelet+TV	Dict-FOCUSS	Tikhonov	PCA

Acceleration R = 3



15.8% RMSE	7.8% RMSE	8.1% RMSE	8.7% RMSE

pdf reconstruction error maps

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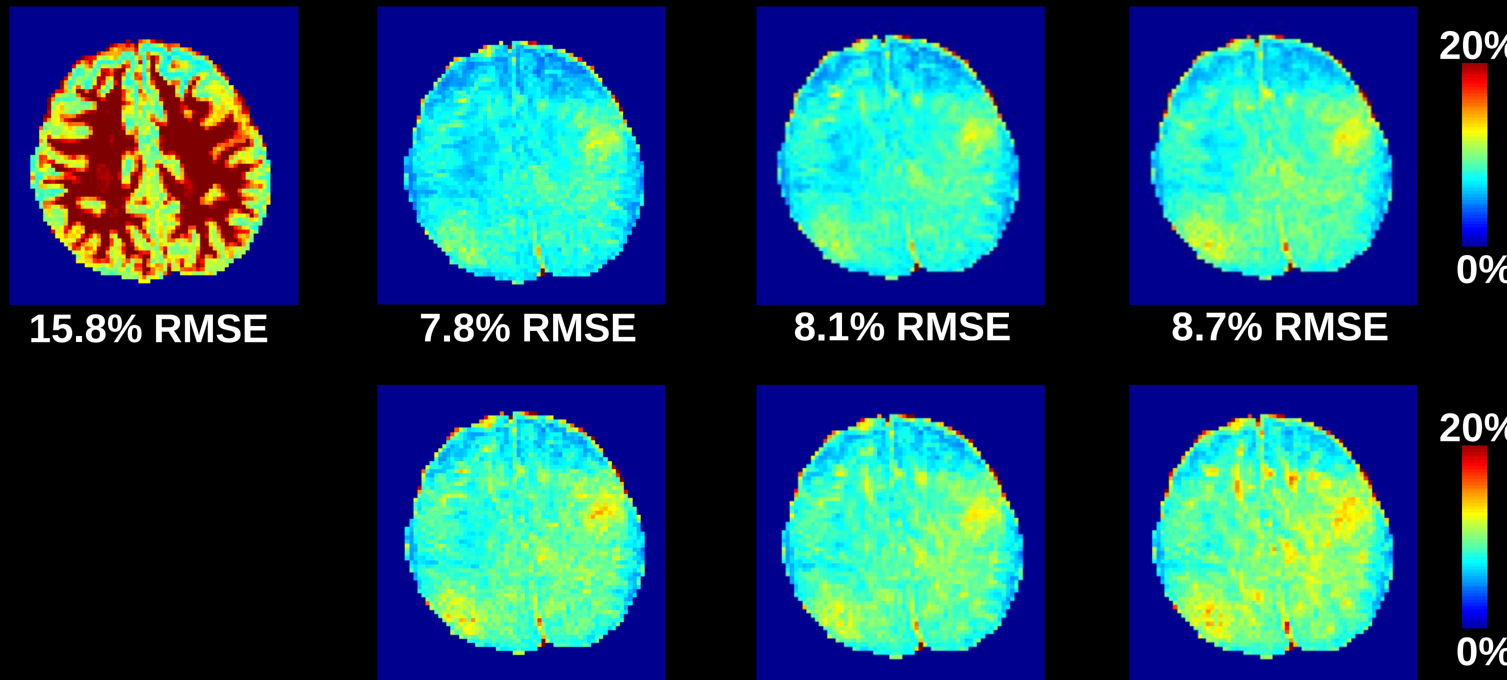
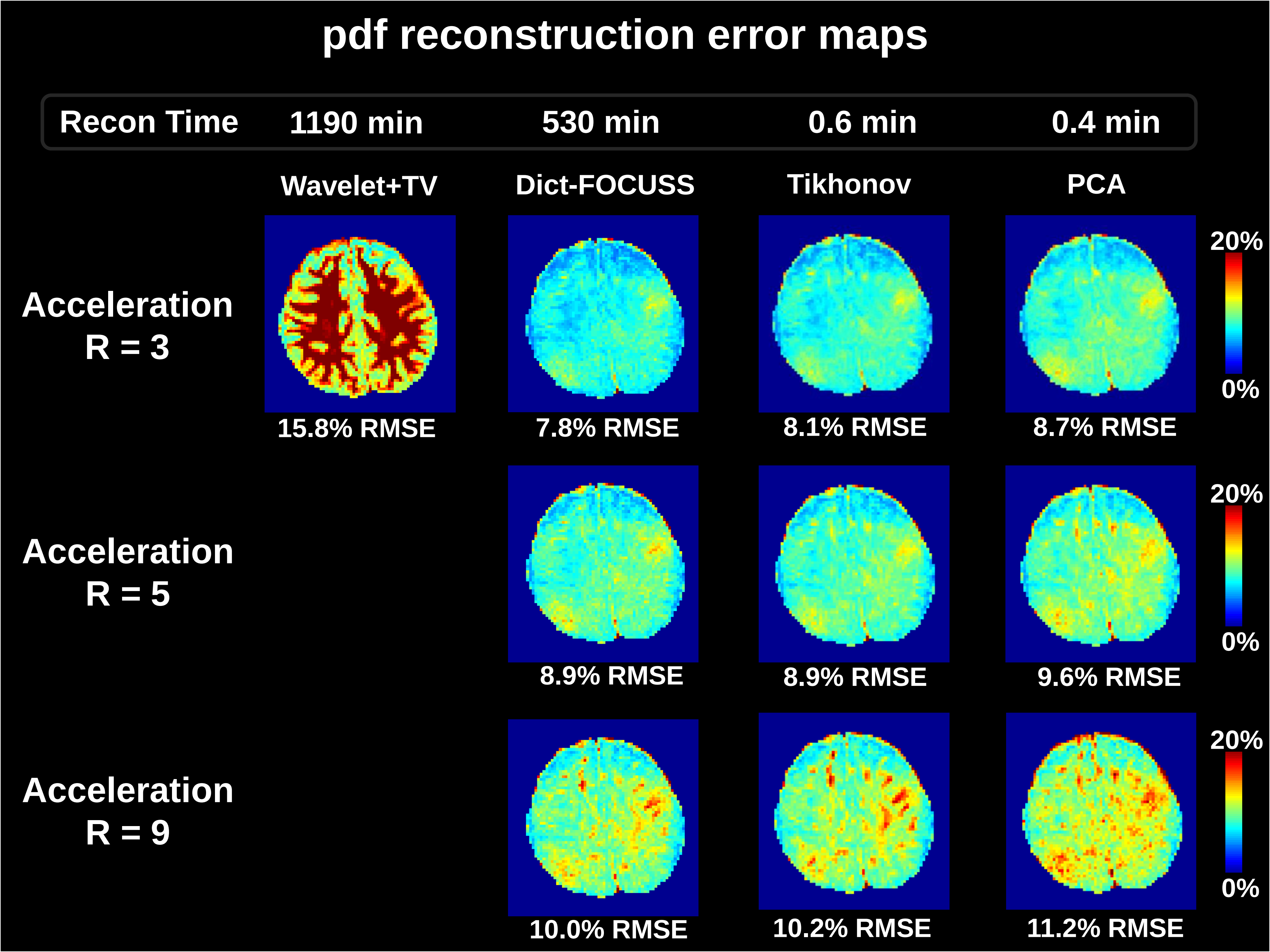
	Wavelet+TV	Dict-FOCUSS	Tikhonov	PCA
Acceleration R = 3	15.8% RMSE	7.8% RMSE	8.1% RMSE	8.7% RMSE
Acceleration R = 5		8.9% RMSE	8.9% RMSE	9.6% RMSE




pdf reconstruction error maps
Recon Time
1190 min
530 min
0.6 min
0.4 min
Wavelet+TV
Dict-FOCUSS
Tikhonov
PCA
Acceleration R = 3
15.8% RMSE
7.8% RMSE
8.1% RMSE
8.7% RMSE
Acceleration R = 5
8.9% RMSE
8.9% RMSE
9.6% RMSE
Acceleration R = 9
10.0% RMSE
10.2% RMSE
11.2% RMSE
20%
0%
20%
0%
20%
0%

Conclusion

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i. <u>Quantitative Susceptibility Mapping</u>

- Closed-form solution:1000-fold speed up obtained relative to state of the art

ii. <u>Diffusion Spectrum Imaging</u>

- Rather than enforcing sparsity, it seems that using a dictionary is the key to good reconstruction
- 1000-fold speed up obtained relative to Compressed Sensing

<u>Software Download:</u>

http://web.mit.edu/berkin/www/software.html